MODEL ANSWERS

Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 3: Statistics & Mechanics

Specimen paper Time: 2 hours	Paper Reference(s) 9MA0/03

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** the questions in Section A and Section B.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 5 questions in this section. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

SECTION A: STATISTICS

Answer ALL questions.

1. *Kaff* coffee is sold in packets. A seller measures the masses of the contents of a random sample of 90 packets of *Kaff* coffee from her stock. The results are shown in the table below.

Mass w (g)	Midpoint y (g)	Frequency f
$240 \le w < 245$	242.5	8
$245 \le w < 248$	246.5	15
$248 \le w < 252$	250.0	35
$252 \le w < 255$	253.5	23
$255 \le w < 260$	257.5	9

(You may use $\sum fy^2 = 5\ 644\ 171.75$)

A histogram is drawn and the class $245 \le w < 248$ is represented by a rectangle of width 1.2 cm and height 10 cm.

(a) Calculate the width and the height of the rectangle representing the class $255 \le w < 260$.
(3)

(b) Use linear interpolation to estimate the median mass of the contents of a packet of *Kaff* coffee to 1 decimal place.
(2)

(c) Estimate the mean and the standard deviation of the mass of the contents of a packet of *Kaff* coffee to 1 decimal place.
(3)

The seller claims that the mean mass of the contents of the packets is more than the stated mass. Given that the stated mass of the contents of a packet of *Kaff* coffee is 250 g and the actual standard deviation of the contents of a packet of *Kaff* coffee is 4 g,

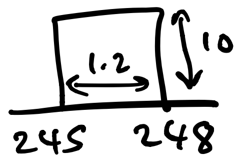
(d) test, using a 5% level of significance, whether or not the seller's claim is justified. State your hypotheses clearly.

(You may assume that the mass of the contents of a packet is normally distributed.)
(5)

(e) Using your answers to parts (b) and (c), comment on the assumption that the mass of the contents of a packet is normally distributed.
(1)

(Total 14 marks)

a)



Area = K × frequency

$(10)(1.2) = K \times 15$

$12 = K \times 15$

$\therefore K = \frac{12}{15} = \frac{4}{5}$ //

For $255 \leq w \leq 260$: Area = K × freq = $\frac{4}{5} \times 9 = \frac{36}{5}$ //

$\therefore$ width × height = $\frac{36}{5}$

$245 \leq w \leq 248$ had a width of 1.2cm.

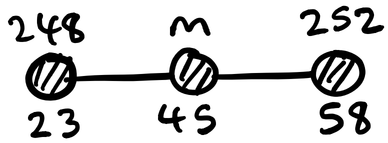
so a class width of 1 has width $\frac{1.2}{3}$ cm

$\therefore$ $255 \leq w \leq 260$ has a width of $5 \times \frac{1.2}{3}$

$= \boxed{2\text{ cm}}$ //

so height = $\frac{\frac{36}{5}}{2} = \boxed{3.6\text{cm}}$

b)



midpoint lies in class $248 \leq w \leq 252$

$$\frac{m - 248}{252 - 248} = \frac{45 - 23}{58 - 23}$$

$$\frac{m - 248}{4} = \frac{22}{35}$$

$$\therefore m - 248 = 4\left(\frac{22}{35}\right)$$

$$m = 248 + 4\left(\frac{22}{35}\right) = \boxed{250.5g}$$

c) mean $= \dfrac{\sum fy}{\sum f} = \dfrac{22535.5}{90} = \boxed{250.4\text{g}}$

$$\text{s.d} = \sqrt{\left(\frac{\sum fy^2}{\sum f}\right) - (250.4)^2} = \sqrt{\frac{5644171.75}{90} - (250.4)^2}$$

$$= \boxed{4.0\text{g}}$$

d) $H_0: \mu = 250$

$H_1: \mu > 250$

$$\bar{X} \sim N\left(250, \frac{4^2}{90}\right)$$

0.171

$$P(\bar{X} > 250.4) = 0.0289\ldots$$

mean of sample, calculated in part c.

$0.171 >$

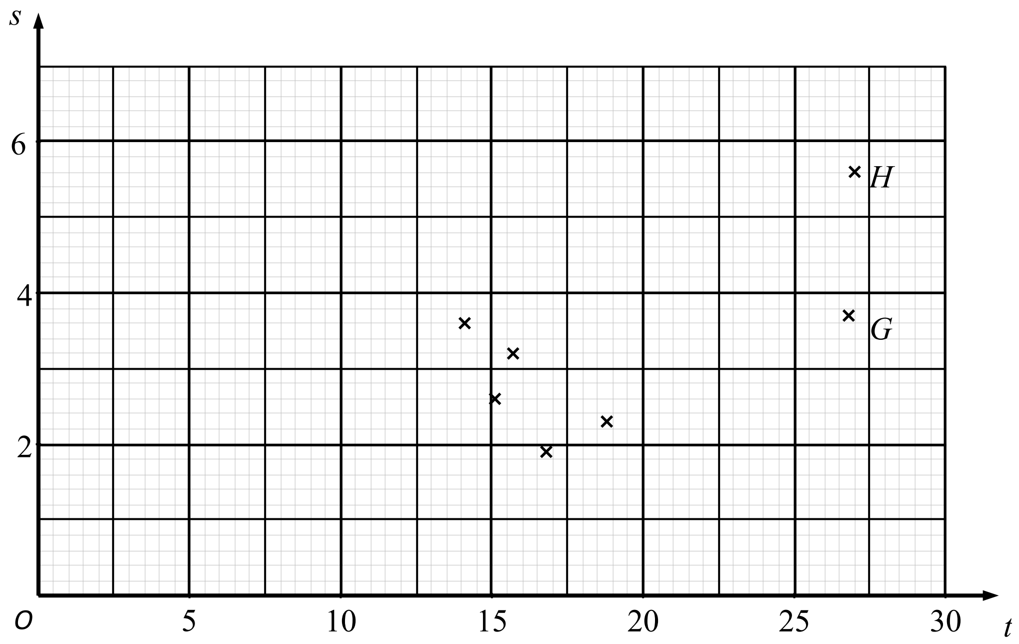
∴ Accept H_0. Insufficient evidence to show that the mean of weight of coffee is greater than 250g.

reject H_0.

Evidence suggests the claim made is correct.

e) mean $\simeq$ median ∴ the assumption that masses are normally distributed is fine.

2. A researcher believes that there is a linear relationship between daily mean temperature and daily total rainfall. The 7 places in the northern hemisphere from the large data set are used. The mean of the daily mean temperatures, t °C, and the mean of the daily total rainfall, s mm, for the month of July in 2015 are shown on the scatter diagram below.



(a) With reference to the scatter diagram, explain why a linear regression model may not be suitable for the relationship between t and s.

(1)

The researcher calculated the product moment correlation coefficient for the 7 places and obtained $r = 0.658$.

(b) Stating your hypotheses clearly, test at the 10% level of significance, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

(c) Using your knowledge of the large data set, suggest the names of the 2 places labelled G and H.

(1)

(d) Using your knowledge from the large data set, and with reference to the locations of the two places labelled G and H, give a reason why these places have the highest temperatures in July.

(2)

(e) Suggest how you could make better use of the large data set to investigate the relationship between daily mean temperature and daily total rainfall.

(1)

(Total 7 marks)

a) Data points don't lie closely to a straight line.

b) $H_0: \rho = 0$ critical value: ± 0.5509
(n = 7, 10%.)

$H_1: \rho > 0$

$0.658 > 0.5509$

$\therefore$ Result is significant. Reject H_0. Evidence suggests that the PMCC is greater than 0.

c) Beijing & Jacksonville.

d) They are closest to the equator.

e) The given data uses information from various places. It would be better if we only used data from one location. Any analysis would be more reliable.

3. For a particular type of bulb, 36% grow into plants with blue flowers and the remainder grow into plants with white flowers. Bulbs are sold in mixed bags of 40

Russell selects a random sample of 5 bags of bulbs.

(a) Find the probability that fewer than 2 of these bags will contain more bulbs that grow into plants with blue flowers than grow into plants with white flowers.

(4)

Maggie takes a random sample of n bulbs.

Using a normal approximation, the probability that more than 244 of these n bulbs will grow into blue flowers is 0.0521 to 4 decimal places.

(b) Find the value of n.

(6)

(Total 10 marks)

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a) $X \sim B[40, 0.36]$, where $X = \#$ of blue bulbs in a bag.

$P(\text{more blue than white in one bag}) = P(X \geq 21)$

$= 1 - P(X \leq 20) = 0.0240$ //

let $Y = \#$ bags out of 5 with more blue than white flowers,

$Y \sim B[5, 0.0240)$

$P(\text{required}) = P(Y \leq 1) = \boxed{0.995}$ (3d.p)

b) $A \sim B[n, 0.36] \simeq N[0.36n, 0.2304n]$

$P(\text{required}) = P(A > 244) = P(A > 244.5)$

applying continuity correction

$= P\left(Z > \frac{244.5 - 0.36n}{\sqrt{0.2304n}}\right) = 0.0521$

using inverse normal function: (area to the left = 0.9479)
$\mu = 0, \sigma = 1$

$$\frac{244.5 - 0.36n}{\sqrt{0.2304n}} = 1.625$$

$$244.5 - 0.36n = (1.625\sqrt{0.2304})\sqrt{n}$$

$$0.36n + 0.78n^{\frac{1}{2}} - 244.5 = 0$$

This is a quadratic in $n^{\frac{1}{2}}$.

<u>using Quadratic Formula</u>:

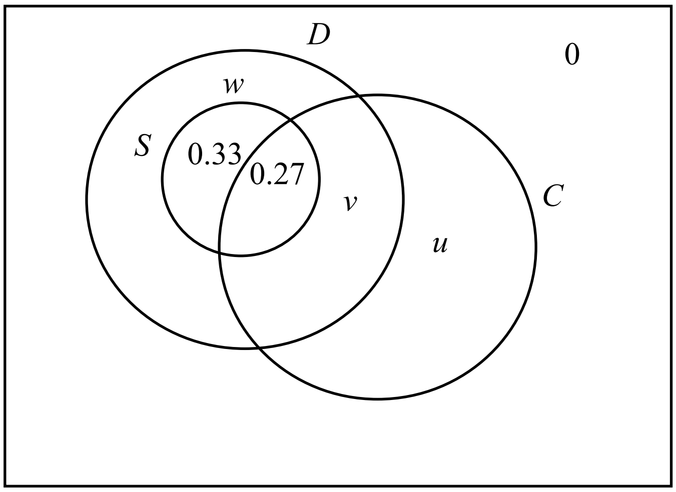
$$\left.\begin{aligned} a &= 0.36 \\ b &= 0.78 \\ c &= -244.5 \end{aligned}\right\} \quad n^{\frac{1}{2}} = 25, \quad \cancel{n^{\frac{1}{2}} = -\frac{163}{6}}$$

not valid.

$$\therefore n = 25^2$$

$$\boxed{n = 625}$$

4. The Venn diagram shows the probabilities of students' lunch boxes containing a drink, sandwiches and a chocolate bar.



D is the event that a lunch box contains a drink,
S is the event that a lunch box contains sandwiches,
C is the event that a lunch box contains a chocolate bar,
u, v and w are probabilities.

(a) Write down $\mathrm{P}(S \cap D')$.

(1)

One day, 80 students each bring in a lunch box.

Given that all 80 lunch boxes contain sandwiches and a drink,

(b) estimate how many of these 80 lunch boxes will contain a chocolate bar.

(3)

Given that the events S and C are independent and that $\mathrm{P}(D \mid C) = \frac{14}{15}$,

(c) calculate the value of u, the value of v and the value of w.

(7)

(Total 11 marks)

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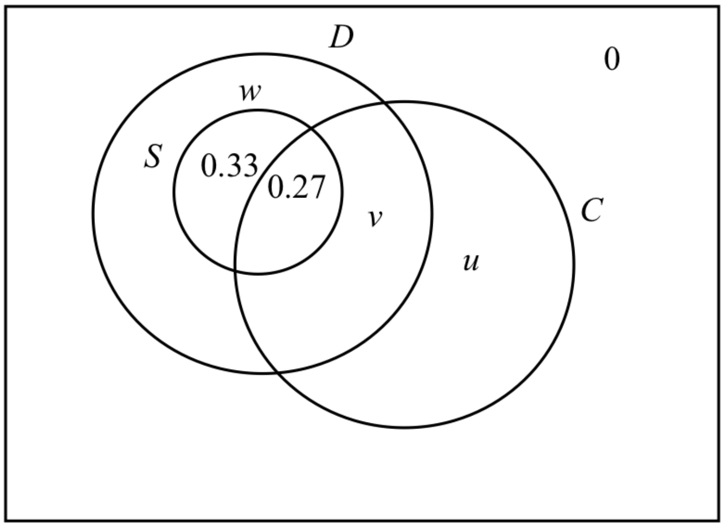
a) $\boxed{0}$ since S is inside D.

b) $P(C \mid S \cap D) = \frac{P(C \cap S \cap D)}{P(S \cap D)} = \frac{0.27}{0.33+0.27} = \frac{9}{20}$

$\therefore$ # lunches with chocolate $= 80 \times \left(\frac{9}{20}\right) = \boxed{36}$

c)



$$P(S) \times P(C) = P(S \cap C) = 0.27$$

$$(0.33 + 0.27) \times (0.27 + v + u) = 0.27$$

$$\therefore \; 0.27 + v + u = \frac{9}{20}$$

$$\text{so} \quad u + v = 0.18 \; // \quad (1)$$

$$P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{14}{15}$$

$$\frac{0.27 + v}{0.27 + v + u} = \frac{14}{15}$$

$$0.27 + v = \frac{14}{15}(0.27) + \frac{14}{15}u + \frac{14}{15}v$$

$$\frac{1}{15}(0.27) + \frac{1}{15}v - \frac{14}{15}u = 0$$

$$0.27 + v - 14u = 0$$

$$14u - v = 0.27 \; // \quad (2)$$

① + ② : $u + \cancel{v} + 14u - \cancel{v} = 0.27 + 0.18$

$15u = \frac{9}{20}$ $\therefore u = \frac{9}{20 \times 15} = \boxed{0.03}$

$\therefore v = 0.18 - 0.03 = \boxed{0.15}$

from ①.

& total probability = 1

$\therefore 0.33 + 0.27 + u + v + w = 1$

$0.6 + 0.15 + 0.03 + w = 1$

$\therefore w = \boxed{0.22}$

5. The lifetimes of batteries sold by company X are normally distributed, with mean 150 hours and standard deviation 25 hours.

A box contains 12 batteries from company X.

(a) Find the expected number of these batteries that have a lifetime of more than 160 hours.
(3)

The lifetimes of batteries sold by company Y are normally distributed, with mean 160 hours and 80% of these batteries have a lifetime of less than 180 hours.

(b) Find the standard deviation of the lifetimes of batteries from company Y.
(3)

Both companies sell their batteries for the same price.

(c) State which company you would recommend. Give reasons for your answer.
(2)

(Total 8 marks)

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a) $L \sim N[150, 25^2]$, where L = lifetime of a battery in hours.

$P(L > 160) = 0.34458...$

$X \sim B[12, 0.34458..]$ where X = # of batteries in a box with a lifetime greater than 160 hours.

$E(X) = np$

$= 12(0.34458..)$

$= \boxed{4.13}$

b) $Y \sim N[160, \sigma^2]$

$P(Y < 180) = 0.8$

$P\left(Z < \frac{180-160}{\sigma}\right) = 0.8$

$P\left(Z < \frac{20}{\sigma}\right) = 0.8$

using inverse normal:

$$\frac{20}{\sigma} = 0.8416$$

$$\therefore \sigma = \frac{20}{0.8416} = \boxed{23.8}$$

c) Y has a greate mean. The standard deviations are relatively close too, so choose Y.

SECTION B: MECHANICS

Answer ALL questions.

1. [*In this question position vectors are given relative to a fixed origin O.*]

A particle P moves with constant acceleration $(\mathbf{i} - 2\mathbf{j})$ m s^{-2}.

At time $t = 0$, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j})$ m and is moving with velocity $\mathbf{u}$ m s^{-1}.

At time $t = 3$ s, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j})$ m.

Find $\mathbf{u}$.

(Total 4 marks)

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2. A particle P moves under the action of a single force in such a way that at time t seconds, where $t \geq 0$, its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = (t^2 - 3t)\,\mathbf{i} - 12t\,\mathbf{j}$$

The mass of P is 0.5 kg.

Find the time at which the magnitude of the force acting on P is 6.5 N.

(Total 7 marks)

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1) $s = -4.5i + 3j$
$u = u$
$v =$
$a = i - 2j$
$t = 3$

$\underline{s = ut + \frac{1}{2}at^2}$:
$-4.5\underline{i} + 3\underline{j} = 3u + \frac{9}{2}(\underline{i} - 2\underline{j})$
$-9\underline{i} + 12\underline{j} = 3u$
$\boxed{\therefore\ u = -3\underline{i} + 4\underline{j}}$

$s = \overrightarrow{AB}$

2) $a = \frac{dv}{dt} = (2t-3)\underline{i} + (-12)\underline{j}$ $\quad m = 0.5kg$

$F = ma = 0.5a = (t - \frac{3}{2})\underline{i} + (-6)\underline{j} = F$

$|F| = \sqrt{(t-\frac{3}{2})^2 + (-6)^2} = 6.5$

$(t-\frac{3}{2})^2 + 36 = 6.5^2 \Rightarrow (t - \frac{3}{2})^2 = \frac{25}{4}$

$\therefore t - \frac{3}{2} = \pm\frac{5}{2}$. $t > 0$ so $\boxed{t = 4}$

3.

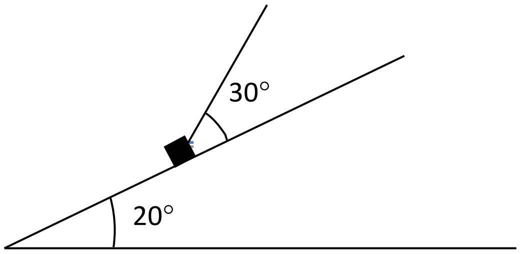

Figure 1

A small box of mass 3 kg moves on a rough plane which is inclined at an angle of 20° to the horizontal. The box is pulled up a line of greatest slope of the plane using a rope which is attached to the box. The rope makes an angle of 30° with the plane, as shown in Figure 1. The rope lies in the vertical plane which contains a line of greatest slope of the plane. The coefficient of friction between the box and the plane is 0.3. The tension in the rope is 25 N.

The box is modelled as a particle, the rope is modelled as a light inextensible string and air resistance is ignored.

Using the model,

(a) find the acceleration of the box.

(7)

(b) Suggest one improvement to the model that would make it more realistic.

(1)

The rope now breaks and the box slows down and comes to rest.

(c) Show that, after the box comes to rest, it immediately starts to move down the plane.

(3)

(Total 11 marks)

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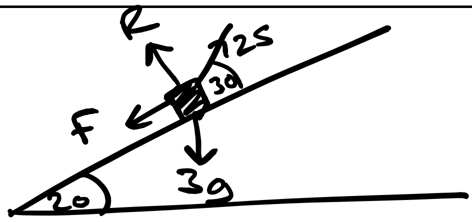
a)



$\nearrow +$ N2L : $25\cos30 - F - 3g\sin20 = 3a$

box is moving ∴ friction is limiting $\Rightarrow F = \mu R = 0.3R$

$R(\nwarrow)$: $R + 25\sin30 = 3g\cos20$

$\therefore R = 3g\cos20 - \frac{25}{2}$ //

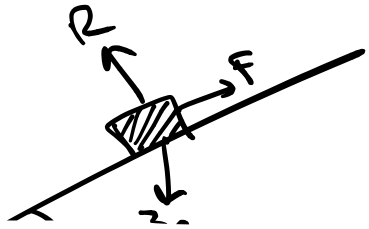
$$\text{so } F = 0.3\left(3g\cos 20 - \frac{25}{2}\right) = 4.54\text{ N}$$

$$\Rightarrow 25\cos 30 - 4.54 - 3g\sin 20 = 3a$$

$$\therefore a = \frac{25\cos 30 - 4.54 - 3g\sin 20}{3} = \boxed{2.35\text{ ms}^{-2}}$$

b) Air resistance should be modelled.

c) consider forces acting on the box when at inst...



N2L

$$\frac{0.3R - 3g\sin 20}{3} = a$$

$R(\nwarrow)$ $F_{max} = 0.3R$

$= 0.9g\cos 20$

$(\swarrow\nearrow)$ $3g\sin 20 \quad 0.9g\cos 20 \quad 9.767 \quad \dfrac{3g\sin 20}{3}$

move

$= -0.30\ldots$

so the box will accelerate down the plane.

4.

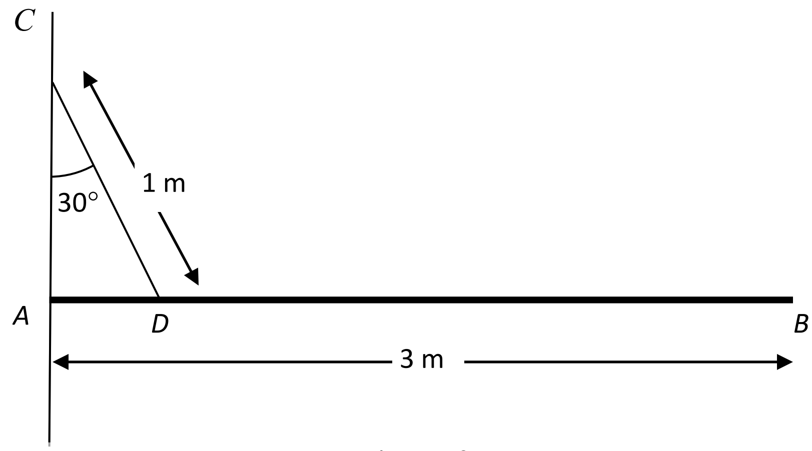


Figure 2

A beam *AB*, of mass 20 kg and length 3 m, is smoothly hinged to a vertical wall at one end *A*.

The beam is held in equilibrium in a horizontal position by a rope of length 1 m. One end of the rope is fixed to a point *C* on the wall which is vertically above *A*. The other end of the rope is fixed to the point *D* on the beam so that angle *ACD* is 30°, as shown in Figure 2. The beam is modelled as a uniform rod and the rope is modelled as a light inextensible string.

Using the model, find

(a) the tension in the rope,

(4)

(b) the direction of the force exerted by the wall on the beam at *A*.

(6)

(c) If the rope were not modelled as being light, state how this would affect the tension in the rope, explaining your answer carefully.

(2)

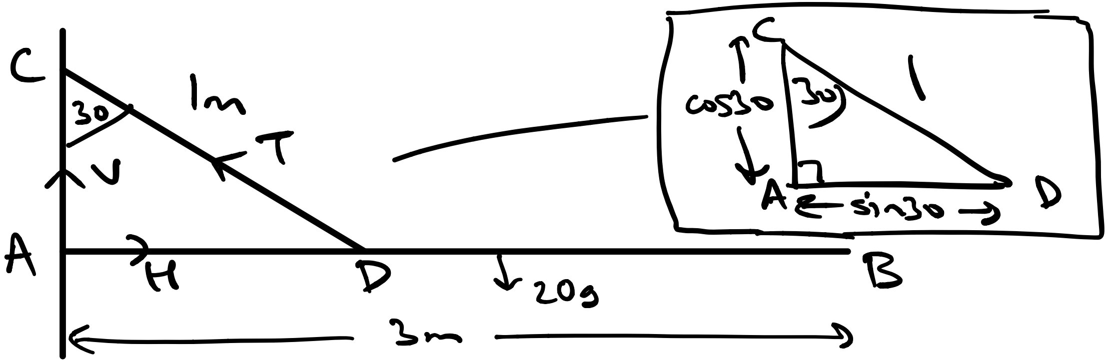
The rope is now removed and replaced by a longer rope which is still attached to the wall at *C* but is now attached to the beam at *G*, where *G* is the midpoint of *AB*. The beam *AB* remains in equilibrium in a horizontal position.

(d) Show that the force exerted by the wall on the beam at *A* now acts horizontally.

(2)

(Total 14 marks)

a)



$\underline{M(A)}$: $T\cos 30\,(\sin 30) = 20g\,(1.5)$

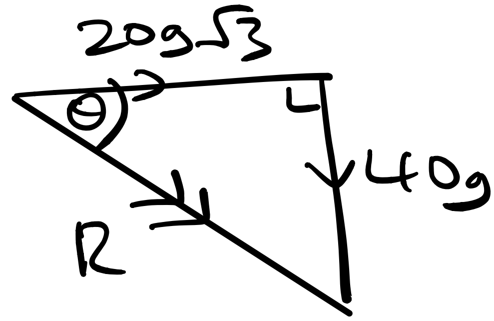
$$T = \frac{20g\,(1.5)}{\sin 30 \cos 30} = \frac{30g}{\frac{\sqrt{3}}{4}} = \boxed{40g\sqrt{3}}$$

b) $R(\updownarrow)$: $T\cos 30 + V = 20g$

$$\therefore V = 20g - T\cos 30 = 20g - 40g\sqrt{3}\cos 30$$
$$= 20g - 60g$$
$$= -40g$$

(so V acts downwards)

$R(\leftrightarrow)$: $H = T\sin 30 = 20g\sqrt{3}$



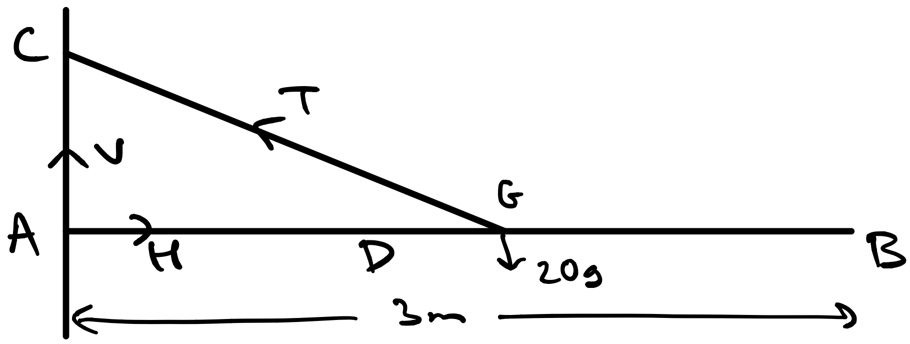
$$\tan\theta = \frac{40g}{20g\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore \theta = \arctan\left(\frac{2}{\sqrt{3}}\right) = \boxed{49.1^\circ}$$

[below the horizontal & away from wall.]

c) Tension will no longer be constant throughout the rope. In fact, tension will increase as we move along the rope from D to C since each 'chunk' of rope needs to support the mass under it

d)



$$M(G): \; V(1.5) = 0$$

$$\therefore V = 0$$

so resultant force acts horizontally.

5.

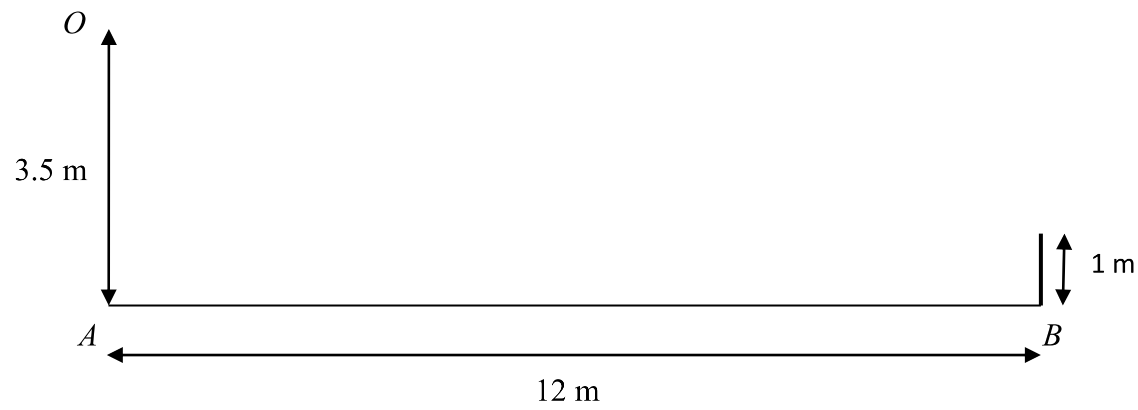


Figure 3

A tennis player serves a ball so as to pass over the net. The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal. The ball is struck at a point O which is 3.5 m vertically above the point A which is on horizontal ground. The bottom of the net is the point B which is on the ground and AB = 12 m. The height of the net is 1 m, as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity. The ball passes over the net at a point which is vertically above B.

Using the model, find

(a) in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net,

(8)

(b) to 2 significant figures, the speed of the ball as it passes over the net.

(4)

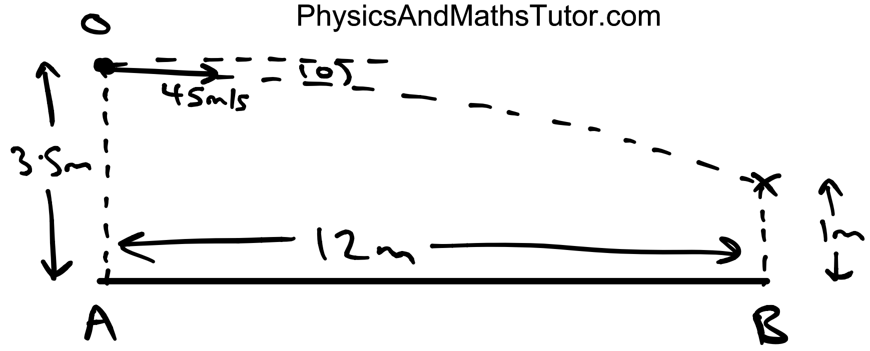
(c) State two limitations of the model that could affect the reliability of your answers.

(2)

(Total 14 marks)

TOTAL FOR PAPER IS 100 MARKS

a)



$\xrightarrow{+}$ $\underline{s = ut}$: $12 = (45\cos 10)t$

$\therefore t = \dfrac{12}{45\cos 10}$

$s = s$
$u = 45\sin 10$
$v =$
$a = g$
$t = \dfrac{12}{45\cos 10}$ $+\downarrow$

$s = ut + \frac{1}{2}at^2$

$s = (45\sin 10)\left(\frac{12}{45\cos 10}\right) + \frac{1}{2}g\left(\frac{12}{45\cos 10}\right)^2$

$= 2.475\text{m}$

$\therefore$ distance above B $= 3.5 - 2.475$
$= 1.025\text{m}$

$\therefore$ distance above net $= 1.025 - 1$

$= 0.025\text{m}$

$= \boxed{2.5\text{cm}}$

b) horizontal speed is constant $= 45\cos 10$

$s =$
$u = 45\sin 10$
$v = v$
$a = g$
$t = \dfrac{12}{45\cos 10}$ $+\downarrow$

$v = u + at = 45\sin 10 + g\left(\frac{12}{45\cos 10}\right)$

$= 10.468\text{m/s}$

$\therefore$ speed $= \sqrt{(45\cos 10)^2 + (10.468)^2}$

$= \boxed{46\text{m/s}}$

c) · we didn't account for air resistance.
· we didn't account for the size of the tennis ball.
· we didn't account for any spin.